1. The probability that Maya will buy a new dress is 0.8,the probability that her friend Sana will buy a new dress is 0.4.If the two events are independent, find the probability that

(6%)

a) Only Maya won't buy a new dress.

$1-0.8=0.2 \rightarrow 0.4\times0.2=$ **0.08**

b) only one will buy a new dress.

$(0.8\times0.6)+(0.4\times2)=0.48+0.08=$ **0.56**

(6)

c) At least one will buy a new dress.

$0.56+(0.8\times0.4)=$ **0.88** ✓

2. Given $P(B)=0.6$, $P(B\cap A')=0.4$ and $P(B/A')=0.8$, find:

(7%)

- $P(A)=$

$P(A'\cap B)$

$=P(B)-P(A\cap B) \rightarrow 0.4=0.6-P(A\cap B)$

$P(A\cap B)=0.2$ ✓

$P(B|A')=\frac{P(B\cap A')}{P(A')}$

$\rightarrow 0.8=\frac{0.4}{P(A')}$

$\rightarrow P(A')=\frac{0.4}{0.8}=0.5$ ✓

$\rightarrow P(A)=1-0.5=$ **0.5**

(7)

- $P(B'\cap A)=$

$P(A)+P(B\cap A)=0.5-0.2$ ✓

$=$ **0.3**

- $P(A'\cap B')=$

$P(A\cup B)'=1-P(A\cup B)$

$=1-(0.5+0.6-0.2)=1-0.9=$ **0.1** ✓

3. Find n if: $4\ {}_nC_2+{}_{n-1}P_1+2n=2$

(6%)

$4\left(\frac{n!}{2(n-2)!}\right)+\left(\frac{(n-1)!}{(n-1-1)!}\right)+2n=2$ (2)

$4\left(\frac{n(n-1)(n-2)!}{2(n-2)!}\right)+\left(\frac{(n-1)(n-2)!}{(n-2)!}\right)+2n=$

(8) $(n(n-1))+(n-1)+2n=2$ (2)

$8(n^2-n)+n-1+2n=2$

$8n^2-8n+n+2n=3$

$8n^2-5n-3=0$ No Solution

(20 %)

4. At a small college , it has been determined that 40% of all students have some type of scholarship support .

- If 20 students are selected at random, what is the probability that :

P: Have scholarship support

$p = 0.4$

$q = 1 - 0.4 = 0.6$

X: Have scholarship support

a) Exactly 6 wouldn't have some type of scholarship support.

$P(4=6) \quad {}^{20}C_6 (0.6)^6 (0.4)^{14}$ (4)

b) At least 2 would have some type of scholarship support.

$P(X \geq 2) = 1 - P(X < 2) = 1 - P(0,1)$

$= 1 - [{}^{20}C_0 (0.4)^0 (0.6)^{20} + {}^{20}C_1 (0.4)^1 (0.6)^{19}]$ (4)

c) No more than 2 wouldn't have some type of scholarship support.

$P(4=0,1,2)$

$P(X \leq 2) = {}^{20}C_0 (0.6)^0 (0.4)^{20} + {}^{20}C_1 (0.6)^1 (0.4)^{19} + {}^{20}C_2 (0.6)^2 (0.4)^{18}$ (4)

d) less than 20 would have some type of scholarship support, given that at most 1 wouldn't have some type of scholarship support.

E F

$E = (X < 20)$

$F = (4 \leq 1) = (X \geq 19)$

$E \cap F = 19 \leq X < 20$

$= X = 19$

$P(E|F) = \dfrac{P(E \cap F)}{P(F)}$

$= \dfrac{{}^{20}C_{19} (0.4)^{19} (0.6)^1}{{}^{20}C_{19} (0.4)^{19} (0.6)^1 + {}^{20}C_{20} (0.4)^{20} (0.6)^0}$ (5)

- In a sample of 600 students what is the expected number of students that have some type of scholarship support .

$\mu = np$

$\mu = 600 \times 40\% = 240$ students (3)

5. A group of 14 tourists arrived to a certain hotel, in how many ways can the guests be assigned to 3 triple , 2 double and one single hotel rooms ?

(3 %)

$\dfrac{14!}{3!3! \times 3!2!1! \times 2!}$ (1)

6. Malda has 10 pairs of shoes, 7 pants and 8 shirts. $5 = 25$

a) In how many ways can she choose one shirt, one pants and a pair of shoes?

(4 %)

${}^8C_1 \times {}^7C_1 \times {}^{10}C_1$

b) In how many ways can she give 6 items away to charity? (4)

${}^{25}C_6$

7. At a university, students are to select 6 representatives for the Student's council. There were 15 names on the ballot paper 8 boys and 7 girls.

(6 %)

a) How many student councils can be formed?

$^{15}C_6 = 5,005$

b) How many student councils can be formed if the number of boys is equal to the number of girls?

$^8C_3 \times {}^7C_3$

c) How many student councils can be formed, if each member is to take up a different responsibility?

$^{15}P_6$

8. Jasmine wants to form a computer password consisting of 7 letters followed by 4 digits.

(8 %)

a) How many possible passwords can she form?

$26 \cdot 26 \cdot 26 \cdot 26 \cdot 26 \cdot 26 \cdot 26 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 26^7 \times 10^4$

b) How many passwords are possible if Jasmine told us that all the letters are distinct and that 1st letter is P and the 5th letter is K?

$\frac{P}{1} \times \frac{24}{25} \times \frac{23}{24} \times \frac{22}{23} \times \frac{21}{22} \times \frac{20}{21} \times \frac{10^4}{10^4} = 24 \times 23 \times 22 \times 21 \times 20 \times 10^4$

c) How many passwords are possible if the 7 letters are the letters of her name (JASMINE) and that the digits are distinct and odd?

$7! \times \frac{5}{5} \times \frac{4}{4} \times \frac{3}{3} \times \frac{2}{2} = 7! \times 5 \times 4 \times 3 \times 2$

9. There are two restaurants A and B in a small town. Suppose that 20% of the people in the town like to dine at neither restaurant. Of those that like to dine in at least one of the two restaurants 50% like to dine at both. If the number of those that like to dine at only A is 3 times the number of those that like to dine at only B, find:

(8 %)

a) $P(A \cup B) = P(A' \cap B') = .20 \rightarrow P(A \cup B)' = .20 \rightarrow 1 - P(A \cup B) = .20 \rightarrow P(A \cup B) = 0.8$

$P(A' \cap B') = 20\% = 0.2$

$P(A \cap B) = 0.5$

$3A = 3B$

b) $P(A) =$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.8 = P(A) + P(B) - 0.5$

$P(B) = 3P(A) \rightarrow \frac{0.8 - 0.5}{4} = \frac{0.3}{4} = 0.075$

$P(A) =$

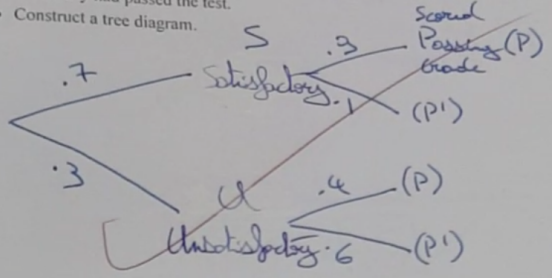
c) $P(A'/B) = \frac{P(A' \cap B)}{P(B)} = \frac{P(B) - P(A \cap B)}{P(B)}$

$= \frac{0.575 + 0.5}{0.575}$

$= 0.13$

10. A corporation uses a selling aptitude test to aid it in the selection of its sales force. Past experience has shown that 70% of all persons applying for a sale position achieved a classification of satisfactory in actual selling. Of those classified as satisfactory, 90% had scored a passing grade on the aptitude test. Only 40% of those classified as unsatisfactory had passed the test. (14 %)

- Construct a tree diagram.



- If one candidate is selected randomly, what is the probability that:

a) He passed the test knowing that he has been classified as unsatisfactory.

$$P(P|U) = \frac{P(P \cap U)}{P(U)} = \frac{(0.3 \times 0.4)}{0.3} = \frac{0.12}{0.3} = \boxed{0.4}$$

b) He got a passing grade on his test.

$$P(P) = (0.7 \times 0.9) + (0.3 \times 0.4)$$
$$= 0.63 + 0.12$$
$$= \boxed{0.75}$$

c) He would be a satisfactory sales person given that he did not get a passing grade on his test?

$$P(S|P') = \frac{P(S \cap P')}{P(P')} = \frac{0.7 \times 0.1}{1 - P(P)} = \frac{0.07}{1 - 0.75}$$
$$= \frac{0.07}{0.25} = \boxed{0.28}$$

d) He didn't get a passing grade or classified as satisfactory.

$$P(P' \cup S) = P(P') + P(S) - P(P' \cap S)$$
$$= (1 - 0.75) + 0.7 - (0.4 \times 0.7)$$
$$= 0.25 + 0.7 - 0.07$$
$$= \boxed{0.88}$$

11. In a bag there are 100 balls : red, green and blue. Half of the balls are with black strips, the others are plain. In that bag, 60% of the balls are not blue and there are the same number of red and green balls, 15 of the red balls have black strips, and a 7/8 of the blue are plain. (18 %)

- Construct a table showing all the information.

Blue : 40 (100 - 60)
Red : 30 ($\frac{60}{2}$)
Green : 30 ($\frac{60}{2}$)

	Have BS (S)	Plain (S')	Total
Blue (B)	5	35	40
Red (R)	15	15	30
Green (G)	30	0	30
Total	50	50	100

- If **one** ball is selected at random what is the probability that the ball is:

a) Red knowing it is plain?

$P(R|P) = \frac{P(R \cap P)}{P(P)} = \frac{15}{50}$

b) Green or with black strips?

$P(G \cup S) = P(G) + P(S) - P(G \cap S)$
$= \frac{30}{100} + \frac{50}{100} - \frac{30}{100} = \frac{50}{100} = \frac{1}{2}$

c) Neither blue nor with black strips?

$P(B' \cap S') = (B \cup S)' = 1 - (B \cup S)$
$= 1 - [\frac{40}{100} + \frac{50}{100} - \frac{5}{100}] = 1 - \frac{85}{100} = \frac{15}{100}$

- If **three** balls are selected at random without replacement ,what is the probability that:

a) They all have the same color?

$\frac{40C_3 + 30C_3 + 30C_3}{100C_3}$

b) They all have different colors?

$\frac{40C_1 + 30C_1 + 30C_1}{100C_3}$

c) Only one is with black strips?

$\frac{50C_1 \times 50C_2}{100C_3}$

d) The 1st two have black strips the 3rd is plain?

$\frac{50P_2 \times 50P_1}{100P_3}$

-1/2

- Are the events "red" and "with black strips" independent? Explain.

$P(R) = \frac{30}{100}$
$P(S) = \frac{50}{100}$] $\rightarrow \frac{30}{100} \times \frac{50}{100} = 0.15 = \frac{15}{100}$
$P(R \cap S) = \frac{15}{100}$ $\rightarrow$ Yes "R" and "S" are independent

- Are the events "green" and "plain" mutually exclusive? Explain.

$P(G \cap P) = 0$

Yes they are mutually exclusive